



A CONCEPT INVENTORY TO IDENTIFY FRACTIONS MISCONCEPTIONS AMONG PROSPECTIVE PRIMARY AND PRESCHOOL TEACHERS IN ROMANIA

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Abstract: The purpose of this article is to validate the relevance of a concept inventory on fractions by measuring the presence and evolution of misconceptions among prospective primary and pre-school teachers, including the overcoming of their misconceptions during and at the end of the instructional intervention. Seven text statements were defined and composed a “Likert scale” concept inventory. This was administered to students at three different stages of the learning process to measure the understanding gain: before the start of the course, after the instructional intervention consisting of a lecture and a seminar on fractions, and finally at the end of the semester. The results from the initial testing confirmed the misconceptions about fractions of the students. Based on the experiment, it is thus obvious the need to allocate a longer time for understanding and fixing the concept of fraction for prospective primary and pre-school teachers.

Key words: Concept inventory, misconceptions, fractions, prospective teachers, primary education, pre-school education, Romania

1. Introduction

The concept of fraction is one of the fundamental concepts of elementary mathematics, which is used in everyday life and on whose knowledge depends more advanced mathematical knowledge. Namkung and Fuchs (2019, p. 36) analyzing the literature finds that “achieving competency with fractions is challenging for many students, and the difficulties associated with learning fractions have been documented widely”. This is because, as many studies have shown, mathematics teachers and prospective teachers have struggled with understanding fractions (e.g., Ma, 2010; Moseley, Okamoto, & Ishida, 2007; Newton, 2008; Olanoff, Lo. & Tobias, 2014; Rodrigues, & Thacker, 2019; Copur-Gencturk, 2021). Therefore, there is a need to improve training on fractions for mathematics teachers, starting with primary school teachers, which can be done after identifying their preconceptions and misconceptions on this concept.

Based on the literature and personal experience, we developed a concept inventory about fractions that was addressed to prospective primary school teachers. This concept inventory was administered in three different stages of the learning process: before the class started, after the instructional intervention which consisted of a lecture and a seminar about fractions and, the last one, at the end of semester. The purpose of this article is to validate the relevance of a concept inventory on fractions by measuring the presence and evolution of misconceptions among prospective primary school teachers, including the overcoming of their misconceptions during and at the end of the instructional intervention.

The research questions are:

1. What are the most common misconceptions about fractions that future primary and secondary school teachers in Romania have?
2. To what extent can misconceptions regarding fractions be overcome by allocating 6 hours of training (theory and practice)?

In Section 2, this article introduces the standardized assessment tool used, the concept inventory, and its development process as envisioned in this study. Considerations regarding fractions and misconceptions

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about this concept are also described. Section 3 is devoted to the research methodology. The results are then presented and discussed in Section 4.

2. Framework

2.1. Concept inventory: definition and building

An increasing number of research studies report tools that measure learning in the context of a course to provide formative assessment of instruction. Among them are the concept inventories, multiple-choice questionnaires inspired by research results and aimed at measuring a learner's knowledge of a concept (or a set of concepts) while capturing the misconceptions he or she has of this concept (Wittie, 2017). It has been applied for misconceptions in a wide range of topics such as: physics (Stoen et al., 2020), chemistry (Schwartz & Barbera, 2014), biology (Kalas et al., 2013), astronomy (Sadler et al., 2010) or even computer science (Taylor et al., 2014). While there are concept inventories in mathematics (Lear, 2019; Stone et al., 2003), they often focus on advanced concepts tested with undergraduate students. To our knowledge, there is no published concept inventory of a basic concept in mathematics education such as the fractions.

It is important for teachers and prospective teachers to be aware of the difficulties that children may have in learning a concept, especially to test the effectiveness of their teaching (Bailey et al., 2012) and to guide future interventions (Taylor et al., 2014). If the pedagogy used is ineffective, there will be little or no learning gain. This does not mean that children will not have learned something, but it may not be what was intended. Regarding assessment, a learner who passes a summative assessment (e.g., an exam) will have learned something, but passing such an assessment is not enough to guarantee the quality of that learning. Concept inventories can in that they are intended to assess the learning gain, and particularly the understanding gain (Sands et al., 2018).

Adams and Wieman establish a six-step process for building a concept inventory (Adams & Wieman, 2011). According to this process, we propose a slightly adjusted process:

- Establish assessment topics through discussions with experienced teachers.
- Identify, through literature and teachers' experience, how learners' thinking about these topics deviates from expert thinking.
- Define text statements around the identified misconceptions and create Likert scale questions based on these ones.
- Administer the questions to learners to validate them.

It is assumed that concept inventories composed of text statements would be more effective in measuring the understanding of concepts and the relationships between them. Indeed, it is recognized in mathematics that identifying an algorithm used in an exercise or a method of problem solving, or observing how the teacher proceeds and reproducing it, are considered easier than giving a personal definition of a concept (Bair, 2017). Therefore, to a lesser degree, positioning oneself in relation to a text statement would be a better measure of understanding than solving a simple exercise.

2.2. The concept of fraction and misconceptions about fractions

The concept of fraction is studied in Romania concentrically through successive returns starting with the 2nd grade of primary education. At the beginning, the fractions are introduced out of necessity and in practical contexts, and the definition is an intuitive one, namely "A part or more of the equal parts into which a whole has been divided is called an ordinary fraction" (Magdaș, 2022).

Subsequently, other aspects related to fractions are introduced, such as operations with fractions (actually, operations with rational numbers), decimal fractions, rational numbers, percentages, etc. Since there is no return to the introduction of fractions in a scientific way, the students form the concept of fraction more through exercises and problems with daily contexts, without the notion of fraction being well structured from a theoretical point of view.

The materials for primary and secondary education related to fractions are numerous and are found in abundance both on the Internet (web pages, videos, practice sheets, tests, educational games, etc.) as

well as in printed works, textbooks and school auxiliaries. The materials addressed to students usually include the explanation of theoretical notions, examples and counterexamples (e.g., Khan Academy, n.d.; SplashLearn, n.d.; SlidePlayer, n.d.; Mathcurious, 2020; etc.). Guides for primary and secondary education teachers represent a very good source of information and training of teachers, usually containing examples, counterexamples and methodological suggestions for approaching fractions (e.g., Nova Scotia Department of Education and Early Childhood Development, 2015; PDST, 2014; Mathematics Navigator, n.d.; etc.). It is very important for teachers to realize that “A global understanding of the concept of fractions and a deep sense of their purposes is more important than learning a set of rules” (PDST, 2014, p. 15).

The literature mentions many misconceptions involving fractions, of which we have selected from Mathematics Navigator (n.d.) some that form the basis of the elaboration of concept inventory for misconceptions about Fractions:

- Students do not understand that when finding fractions of amounts, lengths, or areas, the parts need to be equal in size.
- Students do not understand that fractions are numbers as well as portions of a whole.
- Students think fractions must be less than 1.
- Students restrict interpretation of fractions inappropriately and do not understand that different fractions that name the same amount are equivalent.
- Students think that you cannot convert a fraction to a decimal, that they cannot be compared.
- Students think that a decimal is just two ordinary numbers separated by a dot.

In addition, during the 14 years of teaching the mathematics course for prospective primary and pre-school teachers, the teacher has identified theoretical misconceptions about fractions that the students (in this study, the prospective teachers) have. For example, they often confuse a ratio with a fraction or fail to differentiate between a fraction and a rational number.

3. Research methodology

3.1. Participants

The teaching experiment was attended by third-year students from the Specialization of Pedagogy of Primary and Preschool Education, Babeş-Bolyai University, Cluj-Napoca, Romania. Through this specialization, students are preparing to become primary and pre-school teachers in Romania. Until the time of the experiment, the students did not study in university any mathematics disciplines so that at the initial stage of the experiment their mathematical knowledge were the general knowledge with which they remained after completing the pre-university studies.

3.2. Procedure

The didactical research took place in the first semester of the academic year 2021-2022, between November 2021 and January 2022. The experiment consisted of three tests, namely: an initial test (IT) before studying fractions, at the beginning of November; the Middle Test (MT) which was given immediately after the formative activities were held; and the Final Test (FT) which was given in early January at the end of the recapitulative semester's seminar, that is, about 6 weeks after MT. For the formative activities about the fraction concept were allocated 6 hours of which 4 hours of course and 2 hours of seminar. The course activities were carried out frontally through explanatory exposition, advance question formulation, discussions and debates. The course presented both theoretical aspects, examples and counterexamples for the notion of fraction, along with explanatory diagrams and syntheses. At the seminar were solved exercises with fractions in parallel with the recall of the theory. There were also analyzed statements regarding the fractions asking the students for justifications, examples and counterexamples. The students benefited from course support with the theoretical aspects discussed. During the experiment period, all activities were carried out online.

At the beginning of the semester, students were informed that they would receive additional points for participating in the experiment only if they participated in all three tests. The tests were nominal, the

participating students signed a participation agreement in this regard. Although the number of respondents was 107 at IT, 106 at MT and 112 at FT, but after eliminating the responses of those respondents who did not participate in all three tests, only 103 respondents remained. As a result, the analysis of the answers was made for them.

3.3. Research tool

Based on the experience of the teacher in charge of the mathematics course and the scientific literature, it was established a list of misconceptions about fractions (Table 1).

Table 1. *Misconceptions about fractions*

fr-a	A number written as $\frac{m}{n}$ or m/n , where $n \neq 0$ is an ordinary fraction.
fr-b	An ordinary fraction represents a part or more parts of a whole.
fr-c	A rational number is a fraction.
fr-d	A decimal fraction is a fraction with the denominator power of 10 (10, 100, 1000 etc.)
fr-e	By transforming an ordinary fraction into a decimal fraction, it is obtained a finite number of non-zero decimals.

Starting from these misconceptions, we elaborated the research tool, a theoretical test consisting of seven statements regarding the concept of fraction and related concepts (Table 2). The questions were formulated on a Likert scale with 5 possible answers: “Entirely true”, “True, but to be expanded”, “Partially true, it contains error”, “Entirely false” and “I don't know”. The statements have been formulated so as not to be “Entirely true”. Also, for each item, the respondents had to justify the choice of their answer. IT consisted of 5 items (fr-1 to fr-5 from Table 2), while at MT and FT there were 2 more additional items (fr-6 and fr-7 from Table 2). The tests were conducted using the Google Forms application and were completed online by the respondents. In order to answer the questions, the respondents had the possibility to consult any sources of information (course support, seminar support, consultation with colleagues, internet, etc.) and the time available was long enough (20-30 minutes) for each respondent to have the possibility to complete the questionnaire.

Table 2. *Concept inventory about fractions*

Statements	Source
fr-1. A pair of two numbers m and n written as $\frac{m}{n}$ or m/n , with $n \neq 0$ is an ordinary fraction.	Inspired by fr-a and by the definition of a fraction
fr-2. A ratio $\frac{m}{n}$ between two numbers is an ordinary fraction.	Inspired by fr-a
fr-3. A part or more parts of a whole represent a fraction.	fr-b
fr-4. A rational number is a fraction.	fr-c
fr-5. A decimal fraction is a fraction with the denominator power of 10 (10, 100, 1000 etc.)	fr-d
fr-6. A decimal number is a decimal fraction.	fr-d and fr-e
fr-7. By transforming an ordinary fraction into a decimal fraction, it is obtained a finite number of non-zero decimals.	fr-e

4. Results and discussion

The responses to IT (Figure 1) confirm that students have erroneous preconceptions about fractions. For the first three items, around 70% of the students chose the wrong version “Entirely true”. At fr-4 and fr-5, those who have chosen the wrong answer “Entirely true” is near 50%, but by adding up with the percentages of the answer “I don't know” it is reached that almost 60% of the students have erroneous conceptions or have no formed conception of the respective statements.

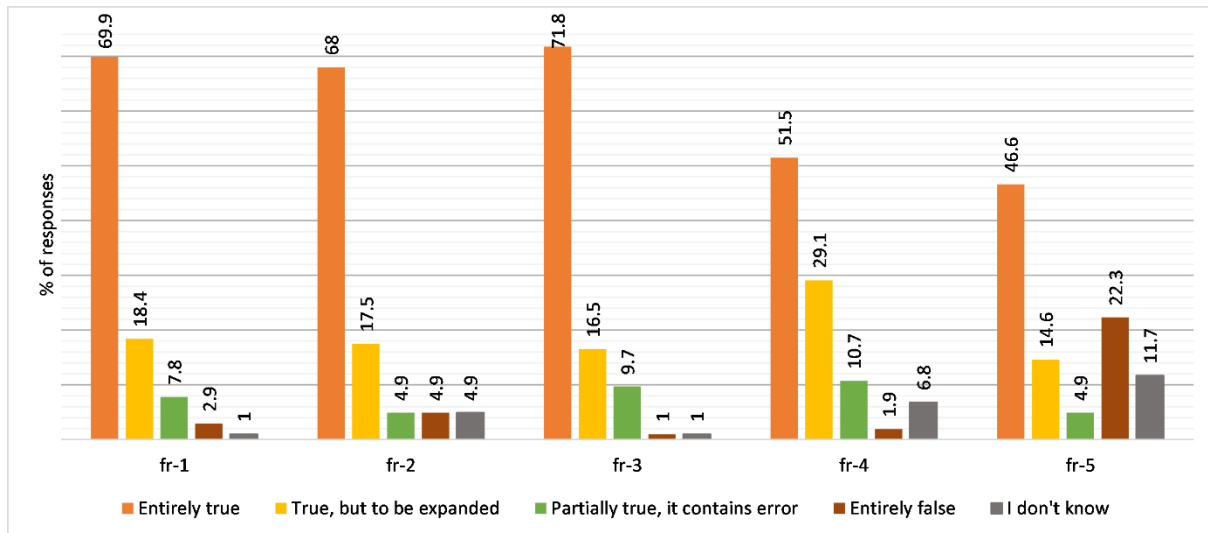


Figure 1. Students' responses to the Initial test (IT)

The responses given by students at MT (Figure 2) show a general decrease in the percentages for the answer “Entirely true”. The largest decreases in the percentages for the answer “Entirely true” took place at fr-2, of 40.8%, and at fr-4, of 22.4%. At fr-3, although the percentage of answers “Entirely true” decreased by 20.3%, it still remains high by over 50%. At fr-5 the decrease was small, below 5%, and we find a polarization of the answers to the extremes, but in this case the answer “Entirely false” is a good one. The smallest decrease in the percentage of people who chose the answer “Entirely true” of only 3.9% occurred in the item fr-1. The items fr-6 and fr-7 were introduced for the first time in MT. Item fr-6 has a high percentage of almost 50% for the answer “Entirely true”, while for item fr-7 the responses are more balanced.

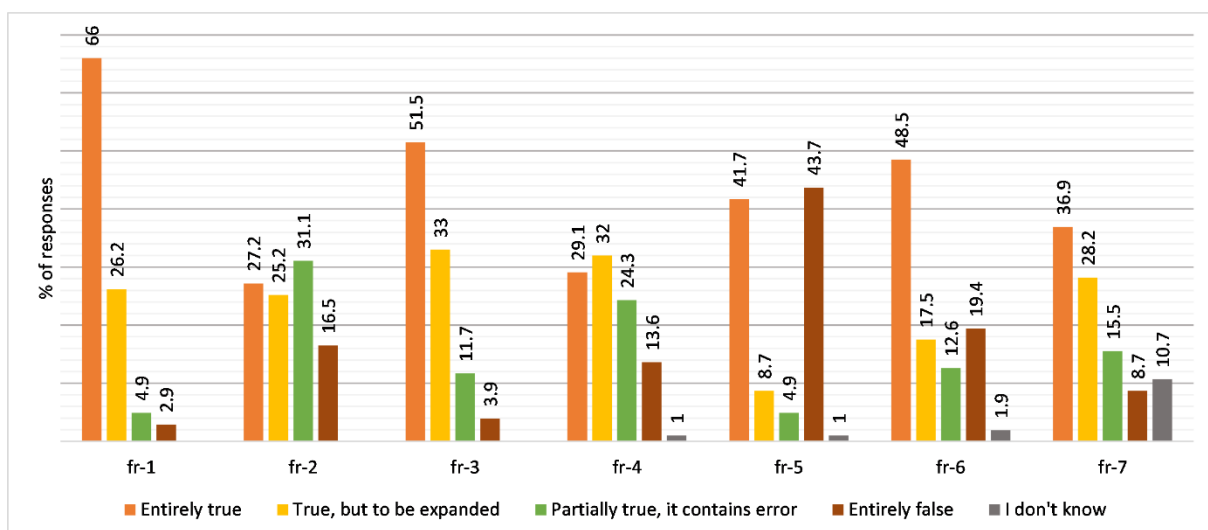


Figure 2. Students' responses to the Middle test (MT)

At FT, the fr-1 still remains at a very high percentage of 68% for “Entirely true” responses and seems to be the most difficult statement. The fr-3 and fr-6 items also have high percentages of over 50% of “Entirely true” responses, they even increased from MT. Greater differences in the choice of answers compared to MT can only be seen in the fr-8 item to which the answer “Entirely true” decreased by almost 12%.

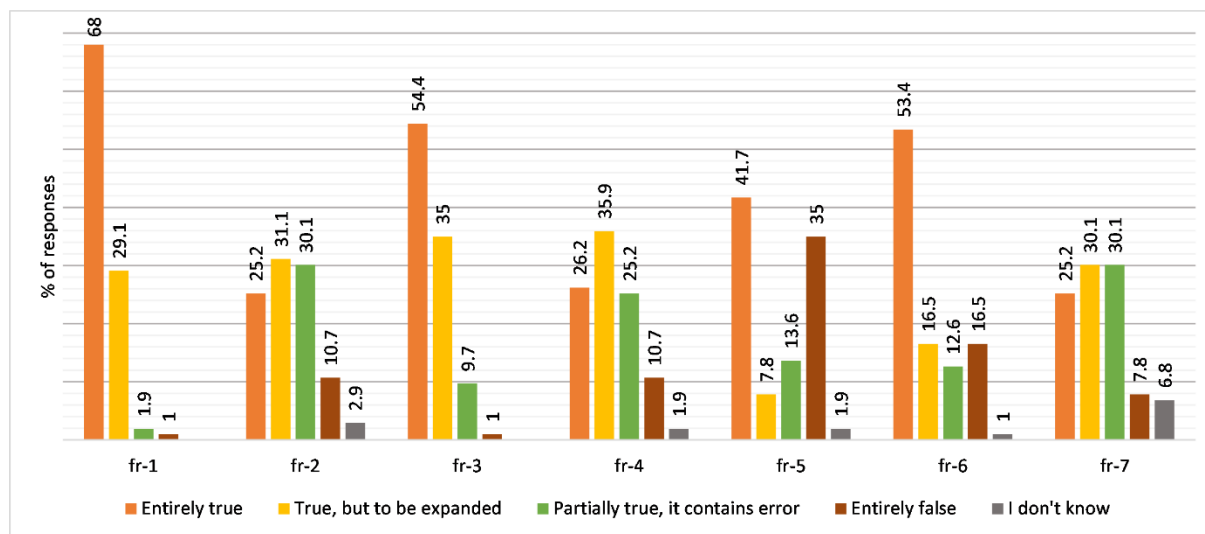


Figure 3. Students' responses to the Final test (FT)

A more complete analysis we will do next, through the individual analysis of the items. In addition to the analysis of the percentages of the answers selected by the respondents, we also analyzed the justifications given by the students. Thus, we found that some respondents, even if they chose the wrong answer, gave a correct justification. So, for the analysis of the answers, we considered for each item both the correct answer and the wrong answers, but with the right justification.

The statement fr-1 is an incomplete scientific definition of an ordinary fraction. The right answer is the statement “True, but to be expanded”, because it is missing the condition that m and n to be integer numbers. From the analysis of the answers to fr-1 (Figure 4(a)) we can see an increase of almost 20% of those who gave the correct answer from IT to FT. At the same time considering the percentage of respondents who gave a correct justification we can say that almost 50% of the respondents know the correct definition of a fraction, although around 7 out of 10 respondents have difficulties in choosing the right answer. Analyzing the answers, we can say that students have difficulty observing the details that make the difference between the correct and complete definition and an incomplete definition.

The statement fr-2 highlights the connection between ratio and ordinary fraction. The right answer is the statement “True, but to be expanded”, because it is missing the condition that m and n to be integer numbers, with $n \neq 0$. By analyzing the answers of the respondents (Figure 4(b)) on this item, an increase in correct answers was obtained by almost 14% from IT to FT. It is observed a very large increase of about 20% in the justification from IT to MT and FT, which shows that respondents, although they understand the link between ratio and ordinary fraction, do not have the ability to choose the most appropriate answer. At the end of the experiment, we can say that almost 6 out of 10 respondents know the connection between ratio and fraction, although more than $2/3$ of them have difficulty choosing the right answer.

The statement fr-3 is an incomplete intuitive definition of a fraction as it is introduced in primary school classes. The right answer is the statement “True, but to be expanded”, because it is missing the condition that the parts in which the whole is divided have to be equal. The complete definition is “A part or more of the equal parts into which a whole has been divided represents a fraction”. The number of correct answers (Figure 4(c)) increased in total by 18.4% from IT to FT. It is also found that the percentage of those who justified the statement correctly but without choosing the right answer increased by 2% from IT to MT and by another 3.8% from MT to FT. This may be due to the students' homework which made them more aware of the fact that the parts into which the whole is divided must be equal.

Concluding, we can say that at the end of the experiment around 6 out of 10 respondents know the empirical definition of a fraction, although nearly $2/3$ have difficulty choosing the right answer.

The fr-4 statement highlights the connection that exists between rational numbers and fractions. The right answer is “Partially true, contains error” because a rational number is not necessarily a fraction by itself but can be expressed as a fraction. For example, the number $1.5/3$ is neither an ordinary nor a decimal fraction, but it is a rational number because it can be expressed as $1/2$, $2/4$, $3/6$, etc. By analyzing the answers of the respondents (Figure 7(d)), we found that it is a high percentage of right justification. Most of the justifications were the following: “a rational number can be written as a fraction”, “true, only that a rational number can be equivalent to several ordinary fractions”, but nevertheless the students chose the answer “Entirely true”. In addition to these, there were other justifications that we did not consider, such as: “fraction is the term we use to denote a rational number”, “the ordinary fraction is called a rational number”, “fractions are part of the set of rational numbers”, “the connection between fractions and rational numbers has a double direction”. We also find that the percentage of correct answers increased with 14.5% from IT to FT, but remains small being selected by around a quarter of respondents. Thus, at the end of the experiment, almost 6 out of 10 respondents knew that there is a connection between rational numbers and fractions, but almost three-quarters of them had difficulty choosing the right answer.

The statement fr-5 is a “Partially true, contains error” definition of a decimal fraction. The statement fr-6 or a similar one as “Decimal fractions are the rational numbers that may be expressed as a fraction whose denominator is a power of ten” can be found in school textbooks, on websites, videos, dictionaries. As a result, students incompletely form the concept of decimal fraction and fail to make the connection between decimal fractions, decimal numbers and ordinary fractions. The correct definition of the decimal fraction is “A fraction expressed by using decimal representation, as opposed to a vulgar fraction” (Clapham & Nicholson, 2014). For the analysis of the answers (Figure 7(e)) we considered two correct answers. The answer “Partially true, contains error” can be considered correct because terminating decimal fractions can be written as a fraction with the denominator power of 10. The “Entirely false” answer can also be considered correct, because a decimal fraction is not an ordinary fraction. Among the correct justifications given by the respondents who chose a wrong answer, we mention: “the decimal fraction is obtained by dividing the numerator by the denominator”, “The decimal fraction can have any denominator”, “The decimal fraction is the result written with a comma for an ordinary fraction”, “The decimal fraction is another way of writing the ordinary fraction, namely with a comma” (note that in the Romania for writing a decimal number it is used the comma instead of the decimal point), “There are also periodic decimal fractions where to the denominator appears the number 9 followed or not by zeros written so many times depends how many numbers are in the period”, “The finite decimal fractions have the denominator power of 10”. This item has the largest increase in correct outputs by 41.4% from IT to MT. Interesting to note is that the total percentage of correct answers remained the same at MT and FT, but the students repositioned themselves in choosing the 2 answers. At the end of the experiment, we can say that almost 6 out of 10 respondents know the definition of a decimal fraction, although over half of them have difficulties in choosing the right answer, which shows once again their uncertainty about this statement, the lack of in-depth fixation of the concept of decimal fraction, requiring a longer time to approach this notion.

The fr-6 statement highlights the link between a decimal number and a decimal fraction. The statement is “Partially true, contains error” because some decimal numbers are decimal fractions, but not all of them. For example, $1.202002000\dots$ has an infinite number of nonrepetitive decimals, so it cannot be transformed in an ordinary fraction, being an irrational number. Among the correct justifications of the students, we mention: “If it has an infinity of decimal places that are not repeated then it is not a decimal fraction”, “Only the numbers that can be transformed into ordinary fractions”, “Any decimal fraction is a decimal number and not the other way”. Analyzing the students’ answers (Figure 7(f)) we can observe that almost 4 out of 10 can make the connection between a decimal number and a fraction, but around three quarters of them have difficulties in choosing the right answer. It is clear that students need a longer time and more exercises to understand and fix this aspect, which is a difficult one.

The statement fr-7 is related to the types of decimal fractions and how to transform an ordinary fraction into a decimal fraction. The right answer is “True, but to be expanded”, because the complete

formulation is “By transforming an ordinary fraction into a decimal fraction it is obtained a finite number of non-zero decimals or an infinite number of pure periodic or ultimately periodic decimals”. The exercises of transformation of an ordinary fraction into a decimal fraction are typical, but from a theoretic point of view analyzing the students’ answers (Figure 7(g)) we find that at the end of the experiment only around 30% of them manage to choose the right answer. Some students gave a justification such as “An infinity of decimal places can also be obtained that is repeated”, while other students gave concrete examples of ordinary fractions, such as $7/9$ or $1/3$. Thus, at the end of the experiment we can say that less than a half of students can make the connection between an ordinary fraction and the type of decimal places of the corresponding decimal fraction.

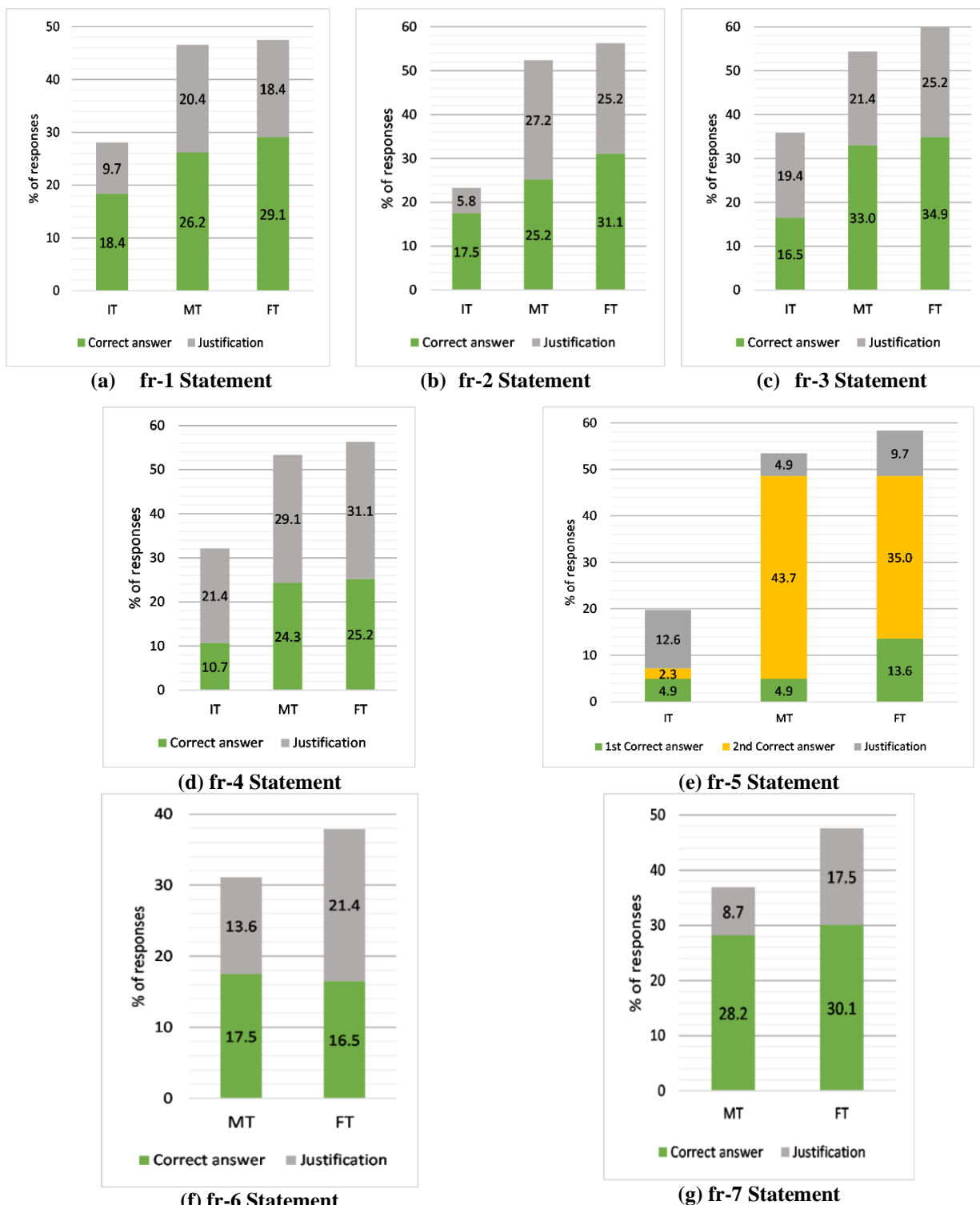


Figure 4. Evolution of the correct answers and justifications for statements

5. Conclusions

In this paper, we presented a study concerning the construction of a concept inventory on fractions in order to identify misconceptions among prospective primary and preschool teachers in Romania. Based on the experience of the teacher in charge of the mathematics course and the scientific literature, five misconceptions were identified. From them, eight text statements were defined and composed a “Likert scale” concept inventory. This was administered to students at three different stages of the learning process to measure the understanding gain: before the start of the course, after the instructional intervention consisting of a lecture and a seminar on fractions, and finally at the end of the semester. The purpose of this article is to validate the relevance of a concept inventory on fractions by measuring the presence and evolution of misconceptions among prospective primary school teachers, including the overcoming of their misconceptions during and at the end of the instructional intervention.

The results from the initial testing confirmed the misconceptions about fractions of the students. Following the formative stage of applying the course and the seminar, certain misconceptions were changed. At the end of the experiment the highest percentage of correct answers, almost 50% were given at fr-5 which allowed the choice of two answers. At the end of the experiment, at four items fr-1, fr-2, fr-3 and fr-7, about 3 out of 10 students answered correctly, and at item fr-6 less than 2 out of 10 students answered correctly. Thus, if at the beginning of the experiment the average of the correct answers to the items was 16.6%, at the end of the experiment, the average percentage of correct answers reached to 30.8%, which shows that almost 15% of the students have overcome preconceptions and misconceptions about fractions. In addition, at the end of the experiment, an average of 21.2% of respondents correctly argued the statements although they had chosen a wrong answer. This fact shows that 1 of 5 students only partially understand the proposed statements and have general knowledge which are not sufficiently well structured so that they have difficulty in choosing the correct answer. Based on the experiment, it is thus obvious the need to allocate a longer time for understanding and fixing the concept of fraction for prospective primary and pre-school teachers, an extremely important concept in the study of mathematics starting with primary education and later in middle school and high school, but also for life.

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