Mental Computation Strategies Used by Preservice Elementary Teachers in Addition with Two-Digit Natural Numbers

Anders Månsson

Abstract: Mental computation strategies in addition on two-digit natural numbers used by preservice elementary teachers are assessed and analyzed in order to examine if in the research literature the mental computation strategies used by the students are adequately categorized. A written test is used to gather mental computation strategies used by preservice elementary teachers. The results indicate a few new strategies used by preservice elementary teachers not already covered by the research literature. On the other hand, the results indicate that there is a theoretical need to clarify some of the definitions of mental computation strategies in research literature.

Key words: addition, mental computation strategy, mental computation test, preservice elementary teacher.

1. Introduction

There have in elementary school research in recent years been an increased interest in mental computation (Hartnett, 2007; Lemonidis et al, 2014). There are several benefits of developing mental computational skills. Mental computation is common in everyday situations (Baranyai et al, 2019a; Thompson, 2010), it increases understanding of elementary calculation rules and the place value system (Gürbüz & Erdem, 2016; Maclellan, 2001), and it enhances number sense (Hajra & Kofman, 2017; Heirdsfield et al, 2011). The mental computation strategies used by elementary school pupils have been extensively researched, but to a lesser degree in relation to preservice elementary teachers (PETs). In this paper PETs’ strategy use is examined and analyzed.

1. 1. Definition of mental computation

There are varying definitions of mental computation in the research literature, and of related concepts or synonyms such as mental math, mental arithmetic, and mental calculation (Lemonidis, 2016; Thompson, 1999). A common characteristic among these definitions is that one calculates without using any equipment (Baranyai et al, 2019b; Lopez, 2014), which is also the definition of mental computation that will be used in this article.

1. 2. Mental computation strategies for addition

The varying ways that arithmetic problems can be solved mentally are called mental computation strategies (Hartnett, 2007; Threlfall, 2000). For instance, 23 + 45 can be calculated by adding the tens and ones separately (20 + 40 = 60 and 3 + 5 = 8) and then adding the sums (60 + 8 = 68). That is a mental computation strategy usually called 1010 (“ten-ten”). Not all strategies are general like this, but some depend on the numbers involved in the calculation (see for example Doubles and near doubles in Table 1). Use of mental computation strategies by elementary school pupils for addition and subtraction with one- and two-digit natural numbers have been extensively studied (Blöte et al, 2000; Heirdsfield, 2001), mental computation for these two operations being in focus of the research more than mental computation strategies in multiplication and division (Callingham, 2005; Heirdsfield et al, 1999). On PETs’ mental computation strategy use there are some studies, for example, in addition and subtraction (Baranyai et al, 2019a; Månsson, 2022a, 2022b), multiplication (Lemonidis et al, 2014; Månsson, 2023; Whitacre, 2007), and division (Mutawah, 2016).

Received April 2024.

Cite as: Månsson, A. (2024). Mental computation strategies used by preservice elementary teachers in addition with two-digit natural numbers, Acta Didactica Napocensia, 17(1), 65-72, https://doi.org/10.24193/adn.17.1.5
An exhaustive search in the research literature resulted in a list of mental computation strategies (see Table 1), which was used in the categorization of the PETs’ explanations. A reference to where the definition can be found in the research literature is given after each strategy. Strategies that are similar are placed next to each other on the table, but other than that in no order.

Table 1. Mental computation strategies for addition on two-digit natural numbers

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto [Automatic calculation]</td>
<td>Retrieve the answer automatically or from memory.</td>
<td>(Lucangeli et al, 2003)</td>
</tr>
</tbody>
</table>
| Counting                         | 3 + 5: 4, 5, 6, 7, 8  
4 + 7: 8, 9, 10, 11  
5 + 12: 7, 9, 11, 13, 15, 17  
5 + 34: 15, 25, 35, 39                                                                 | (McIntosh & Dole, 2005) (Thompson, 1999) (McIntosh & Dole, 2005) |
| Commutativity                    | 2 + 13 = 13 + 2                                                                                | (McIntosh & Dole, 2005)                    |
| Doubles and near doubles         | 6 + 7: 6 + 6 is 12, so it is one more.  
15 + 17: 15 + 15 + 2 = 30 + 2 = 32  
8 + 5 = 13 since 8 and 8 is 16...take away 3.                                                 | (McIntosh & Dole, 2005) (Cooper et al, 1995) (Thompson, 1999) |
| Using tens as the unit           | 80 + 50 = 8 tens + 5 tens = 13 tens = 130                                                   | (McIntosh & Dole, 2005)                    |
| SA [standard algorithm done mentally] | Mental image of pen and paper algorithm, placing numbers under each other, as on paper, and carrying out the operation, right to left. | (Heirdsfield, 2001)                         |
| 1010 [decomposition, regrouping, splitting, partitioning] | 46 + 23 → 40 + 20 = 60 → 6 + 3 = 9 → 60 + 9 = 69  
38 + 16 → 30 + 10 = 40 → 8 + 6 = 14 → 40 + 10 = 50 → 50 + 4 = 54  
26 + 38 → (20 + 6) + (30 + 8) = (20 + 30) + (6 + 8) = 50 + (6 + 4) + 4 = 64                                                                 | (Beishuizen, 1993) (Cooper et al, 1995) |
| u-1010 [1010 right to left]      | 28 + 35 → 8 + 5 = 13 → 20 + 30 = 50 → 13 + 50 = 63  
46 + 23 → 6 + 3 = 9 → 40 + 20 = 60 → 60 + 9 = 69  
26 + 38 → 6 + 8 = 14 → 20 + 30 + 10 = 60 → 64                                                                 | (Joung, 2018) (Beishuizen, 1993) (Cooper et al, 1995) |
| 10s [1010 stepwise, cumulative sum] | 45 + 39 = ((40 + 30) + 5) + 9 = (70 + 5) + 9 = 75 + 9 = 84                                     | (Reys et al, 1995)                         |
| Cumulo-partial sum strategies     | 46 + 57 → 40 + 50 = 90 → 90 + 6 = 96 → 96 + 4 = 100 → 100 + 3 = 103                           | (Threlfall, 1998)                          |
| N10 [stringing, jumping, sequencing, cumulative] | 46 + 23 → 46 + 20 = 66 → 66 + 3 = 69  
46 + 23 → 40 + 23 = 63 → 63 + 6 = 69  
38 + 16 → 38 + 10 = 48 → 48 + 2 + 4 = 54  
| Cumulative sum strategies         | 46 + 57 → 46 + 50 = 96 → 96 + 4 = 100 → 100 + 3 = 103                                        | (Threlfall, 1998)                          |
| u-N10 [N10 right to left]         | 46 + 23 → 46 + 3 = 49 → 49 + 20 = 69                                                         | (Beishuizen, 1993)                         |
| N10C [stringing with compensation] | 52 + 79 → 52 + 80 = 132 → 132 − 1 = 131  
45 + 39 → 50 + 39 = 89 → 89 − 5 = 84  
45 + 39 → 50 + 40 = 90 → 90 − 5 − 1 = 90 − 6 = 84                                                                 | (Baranyai et al, 2019) (Reys et al, 1995) |
| A10 [adding-on, bridging through ten] | 35 + 29 = (35 + 5) + 24 = 40 + 24 = 64                                                       | (Blöte et al, 2000)                        |
| Decomposition                    | 72 + 37 → 72 + 28 = 100 → 100 + 9 = 109                                                       | (Cooper et al, 1995)                       |
| C10 [Formation of units of 10]   | 43 + 6 = (43 + 7) − 1                                                                        | (Lucangeli et al, 2003)                    |
### 1. 3. Teaching mental computation strategies in school

Learning different mental computation strategies and knowing when to use them is important to be able to do mental computation efficiently (Hajra & Kofman, 2017; Thompson, 2010). Some strategies pupils can discover on their own, but this is not true for all pupils nor strategies (Murphy, 2004). Research points out that pupils are not sufficiently showed mental computation strategies in education (McIntosh et al, 1995). That can lead to some pupils using inefficient mental computation strategies, such as use of standard written methods in mental computation (Baranyai et al, 2019b; Joung, 2018). Research shows that pupils that have more refined skills use a range of mental strategies (Hope & Sherrill, 1987). Because the traditional teaching methods do not satisfactorily improve pupils’ numeracy proficiency, some researchers stress the importance of teaching mental calculation strategies (Thompson, 2009). One reason for teachers not doing so is their own lack of knowledge of suitable mental computation strategies (Hartnett, 2007). As the next generation of teachers PETs need a strong foundation of mental computation (Heirdsfield & Cooper, 2004; Lemonidis et al, 2014; Whitacre, 2007).

### 2. Method

#### 2.1. Research aim and question

Knowing which strategies PETs use is important for continued professional development and in research on mental computation (Heirdsfield & Lamb, 2005; Valenta & Enge, 2013). The aim of the research presented in this paper is to study the mental computation strategies used by PETs and to check if these strategies are covered by the existing research literature. For this purpose, a written test was used, which was indicated to be a reliable and useful method in Månsson (2022a). There is no paper containing a complete and exhaustive list of all the mental computation strategies found in the research literature. Further, the definitions vary and do not necessarily elucidate all possible strategies. Research on mental computation has mostly been done using elementary school pupils. However, it is not evident that pupils use the same mental strategies as PETs. Because PETs have more mathematical schooling than elementary school pupils it is conceivable that they use another set of strategies than pupils do.

The research question investigated in this paper is:

In addition to the strategies documented in existing literature, what additional approaches do preservice elementary teachers employ when performing mental computation on two-digit natural numbers?

The answer to this question can indicate that may be a need to extend and improve the list of mental computation strategies found in research literature (Table 1). The reason for limiting the research to two-digit numbers is because they are more interesting than one-digit numbers when it comes to developing number sense and flexible number operations (Beishuizen et al, 1997; McIntosh et al, 1992).

The research was carried out in 2021.

#### 2.2. Research participants

163 (48 first, 54 second- and 61 third-year) PETs from two different mid-sized universities in Norway were involved in the research. Their participation was voluntary and anonymous. The PETs were not given any schooling on mental computation strategies before doing the test.
2.3. Research instrument

The PETs’ mental computation strategy use was measured with a written test containing 15 additions (Figure 1):

1. 11 + 13  
2. 17 + 18  
3. 62 + 27  
4. 80 + 50  
5. 76 + 58  
6. 44 + 33  
7. 38 + 76  
8. 60 + 37  
9. 47 + 45  
10. 64 + 46  
11. 70 + 67  
12. 97 + 86  
13. 88 + 88  
14. 45 + 79  
15. 99 + 98

Figure 1. Operations included in the test

The operations were selected so that several different strategies would be used and induced by the PETs, with the ambition that these operations at least could induce the strategies included in Table 1. Figure 2 contains the test instructions and shows how the operations were presented.

**Important instructions! For every operation do these steps in order:**

1. Calculate the result of the operation in your head.
   - The calculation must be done solely in your head. You are **not** allowed to write anything on the paper.
   - If you do not know the answer after you have been thinking for a while, do not write anything and instead go to the next operation.

2. Write down the answer.

3. Explain mathematically how you were thinking.

**Operation 1**

11 + 13 = ______

Explanation: __________________________________________________________
   __________________________________________________________

Figure 2. Test instructions together with the first operation

Based on the PET’s written explanations their strategy use was categorized by the author using Table 1. In Månsson (2022a) this was indicated to be a reliable and informative approach. Basing the categorizations on the PETs’ written explanations is an operational approach, where one is not speculating on how they were “really thinking” when they calculated.

2.4. Data collection

The PETs were not informed of the test beforehand, so they had no way of preparing for it. There was no given a time limit to the test. They were instructed to calculate each operation mentally, then write down the answer with an explanation on how they solved the operation (see Figure 2). The author categorized students’ explanations based on which mental computation strategy from Table 1 they had used.

2.5. Results

In this section is presented and commented on instances where the PETs used strategies that differed strategies in Table 1, which answers the research question in section 2.1.
2.5.1. **Strategy combinations.** It was not uncommon for the PETs to use combinations of the strategies in Table 1. For example, one PET calculated exercise 2 (17 + 18) as “(10 + 10) + (8 + 2 + 5)”. This could be considered as 1010 since it is like the third defining example of 1010 in Table 1:

\[ 26 + 38 \rightarrow (20 + 6) + (30 + 8) = (20 + 30) + (6 + 8) = 50 + (6 + 4) + 4 = 64 \]

However, it could also be considered as a combination of 1010 and A10. The question of whether combinations of strategies should be treated as distinct strategies is subject to debate, as it could potentially result in many different and detailed strategies. Then it is perhaps better to group similar strategies, as in Månsson (2022a), since they are hard to separate anyway.

2.5.2. **Unclear strategy use.** In some cases, it is hard to assess exactly which strategy a PET has used. For example, in exercise 9 (47 + 45) a PET gave the explanation: “45 + 45 = 90 + 2 = 92”. Here it is not possible to say with certainty if the PET was using the strategy Round to multiples of five, Doubles and near doubles, Decomposition, or some other strategy. Having access only to the PET’s written explanation the author categorized his or her strategy use as unclear. Another similar example is exercise 2 (17 + 18), which one PET calculated as “15 + 15 = 30 with 2 + 3 = 5 left over”, or exercise 6 (44 + 33) that another PET calculated “33 + 33 = 66 and 66 + 11 = 77”. This could be interpreted as Doubles and near doubles, but it could also be interpreted as a separate strategy, which would be to “take the double of the smallest of the two numbers and then add the difference”. This would be the same as Doubles and near doubles but without restricting doubles with the word “near” since one could use this strategy even if the numbers are not near.

2.5.3. **Overlap of SA, 1010, u-1010, and single-digit manipulations.** Some PETs’ explanations are overlaps of the strategies SA (doing the standard algorithm mentally) and u-1010 (adding first the ones and then the tens). In practice they do the steps in SA but without explicitly placing numbers vertically under one another. For example, one PET explained his or her calculation of exercise 2 (17 + 18) in the following way: “I add the last digits in each number, and then the first digits where I get 1 in memory in the calculation of the first digits.” There are also instances where they do single-digit addition as in SA but starting with the ten digits. For example, in exercise 7 (38 + 76) one PET calculated: “Sees that 8 + 6 becomes over 10 so thinks 3 + 7 + 1 = 11 and then 8 + 6 = 14 remembering that the 1 in 14 have already been accounted for”. This could be considered a separate strategy from SA, 1010 and u-1010, where one does single-digit manipulations while observing and remembering to add ten exchanges. This is also the case in the following PET explanation of exercise 6 (44 + 33): “4 + 3 = 7 two times”. It is unclear if it is SA, 1010, u-1010, or a case of “single-digit manipulation”. Since the strategies SA, 1010, u-1010, and “single-digit-manipulations”, are similar and difficult to distinguish a practical approach is to group them and consider them as one strategy.

2.5.4. **Adding numbers both ending with zeroes.** A common strategy used in exercise 4 (80 + 50) is to do the single-digit calculation 8 + 5 = 13 and then add on a zero. This is not a strategy contained in Table 1. One could call it “Single-digit manipulation and adding on zeroes”. It is like Using tens as the unit, but this strategy had no instances of among the PETs’ explanations. Another strategy used in exercise 4 by some PETs was 50 + 50 = 100, 100 + 30“. It is unclear if this strategy is Decomposition, Doubles or near doubles, a variant of A10, or a new strategy where one bridges through hundreds instead of tens.

2.5.5. **Hundreds, closest ten, or multiples of tens?** In exercise 15 (99 + 98) one PET gave the explanation: “100 + 100 = 200 − 1 − 2 = 197”. Here it is not possible to say if the PET had in mind a strategy rounding off to the closest ten, to some multiple of ten, or to (the closest) hundreds. If the exercise instead would have been 89 + 88, and the explanation would have been “100 + 100 = 200 − 11 − 12”, then it would be a case of rounding off to multiples of tens or to hundreds, which are strategies not explicitly described in the list of strategies, although they are like for example A10 and Decomposition. If the explanation to 89 + 88 instead would have been “90 + 90 = 180 − 1 − 2”, then one could rule out the rounding off to hundreds. And if the exercise would have been 69 + 68 and the explanation “70 + 80 = 150 − 13” then one could rule out both rounding off to hundreds and the closest ten. In any case all these strategies are similar and have a similar core idea.
3. Discussion and conclusions

Most of the strategic ideas the PETs used have already been found in the research literature, that is in the list of strategies in Table 1. There were no clear instances where one could say with certainty that a PET used a strategy which is not covered, or does not overlap with some strategy, in the list. The answer to the research question is therefore that there are no, or few strategies used by PETs not contained in the scientific literature. PETs therefore seem to use the same spectrum of strategies for addition as elementary school pupils.

There were instances where it is uncertain which strategy in the list the PET used, or if they used a combination of strategies in the list, or if they used a strategy not part of the list at all. One reason for this uncertainty is found in the PETs’ sometimes ambiguously written explanations, as well as overlap between the strategy definitions. Some of the strategies in research literature are not defined precisely enough for one to be able to categorize explanations unambiguously. Often the strategies are defined only by giving examples, which can be a problem when categorizing. For instance, if defining Bridging through ten (A10) by giving the example

\[ 34 + 19 = 34 + 6 + 13 = 40 + 13 = 53 \]

then does the calculation

\[ 34 + 19 = 34 + 16 + 3 = 50 + 3 = 53 \]

also qualify as a Bridging through ten even though it does not bridge through the closest multiple of ten? Or, as another example, where does one draw the line between doing u-1010 and SA? If the calculational steps are exactly as in the SA, but one does not mentally perceive the numbers under each other vertically, is it then a case of u-1010 or SA, or is it a separate strategy, that one could call “Single-digit manipulation”? When categorizing, one possibility is to group similar strategies such as 1010, u-1010 and SA, as was done in Månsson (2022a). In this case, it would not matter as much if there were overlaps between similar strategies. In other situations, however, one may be interested in differentiating between similar strategies and then it could be a problem with overlapping and unclearly defined strategies. The list of strategies in Table 1 thus seems exhaustive, but there is a theoretical need to clarify the strategies, making them more precise and less overlapping.

References


Beishuizen, M., Van Putten, C.M., & Van Mulken, F. (1997). Mental arithmetic and strategy use with indirect number problems up to one hundred. Learning and Instruction, 7, 87-106.


Månsson, A. (2022a). Categorization reliability of preservice elementary teachers’ use of mental computation addition strategies on natural numbers using a written questionnaire. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bolzano (en ligne), Italy.


Authors
Anders MÅNSSON, Oslo Metropolitan University, Oslo (Norway). E-mail: andersm@oslomet.no.